Physics of Fluids (NAPF-12)
July 2, 2013, 14.00 – 17.00 uur

This is a closed-book exam; only simple pocket calculators are allowed.
Put your name and s-number on every sheet that you hand in.

Question 1 (30 p)

The conservation of mass statement for a material volume *V(t)* in a flowing fluid can be written as

$$\frac{d}{dt}\int_{V(t)} \rho(\mathbf{x},t)dV = 0.$$

Similarly, Newton's second law for a material volume *V(t)* in a flowing fluid can be written as

$$\frac{d}{dt}\int_{V(t)} \rho(\mathbf{x},t)\mathbf{u}(\mathbf{x},t)dV = \int_{V(t)} \rho(\mathbf{x},t)\mathbf{g}dV + \int_{A(t)} \mathbf{f}(\mathbf{n},\mathbf{x},t)dA,$$

with *A(t)* the surface of *V(t)*.

15p a) Use these integral equations as a starting point to derive the differential equation of mass (the continuity equation)

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0$$

and Cauchy's equation of motion

$$\rho\frac{Du_j}{Dt} = \rho g_j + \frac{\partial}{\partial x_i}\left(\tau_{ij}\right)$$

by using Reynolds transport theorem and Gauss divergence theorem.

8p b) The Navier-Stokes momentum equation for an incompressible isotropic Newtonian fluid reads:

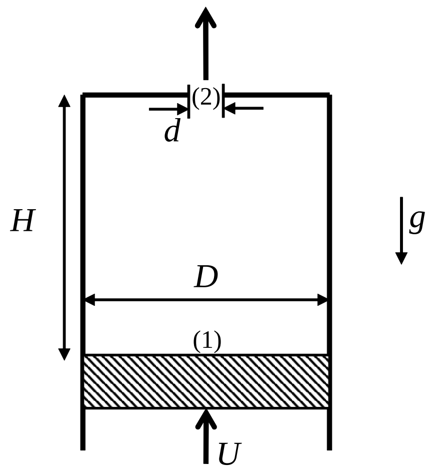
$$\rho\frac{D\mathbf{u}}{Dt} = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{u}$$

Let's analyze a general flow situation of such an incompressible fluid having density ρ and viscosity μ, where the flow field has a characteristic length scale L, a characteristic velocity scale U and a frequency Ω. Use these to define dimensionless parameters for x_i, u_i, t, etc. and write the Navier-Stokes momentum equation in dimensionless form. Show that the flow can be characterized by 3 dimensionless numbers (Strouhal, Reynolds and Froude number). Describe the physical meaning of these numbers.

7p c) Alternatively, Buckingham's Π-theorem can be used to derive dimensionless numbers directly from the 6 system parameters ρ, μ, L, U, Ω, g. Do this. How are these Π-terms related to the Strouhal number, Reynolds number and Froude number?)

Question 2 (30p)

The figure below shows a schematic representation of a vertical syringe (NL: injectiespuit) filled with a fluid of density $\rho = 1000$ kg/m^3. The piston (NL: zuiger) moves upwards with a constant speed U = 1 cm/s. The syringe has diameter D = 16



mm, the diameter of the syringe exit is d = 2 mm and H = 20 mm. The pressure at the exit, point (2), is equal to the atmospheric pressure (p_{atm} = 101.3 kPa). The acceleration of gravity g points vertically downward. The fluid can be assumed to be incompressible and viscosity effects can be neglected.

5p a) The pressure difference Δp between point (1) right above the piston and point (2) at the syringe exit is a function of H, D, U, d, ρ and g. Use Buckingham's Π-theorem to derive a set of dimensionless Π-terms that describes this dependence.

3p b) Calculate the velocity U_2 and volumetric flow rate Q of the fluid at the exit (point (2)).

5p c) Calculate the pressure difference Δp.

3p d) What is the pressure at point (1) right above the piston? Determine the force exerted by the fluid on the piston.

6p e) The solution of this problem can be written in terms of dimensionless Π-terms as $\Pi_1 = \Phi(\Pi_2, \Pi_3, ..., \Pi_n)$. Use the Π-terms derived at a) and determine Φ. [Hint: start by writing your answer at c) for Δp as a function of H, D, U, d, ρ and g].

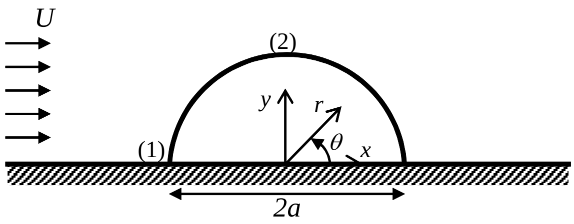
Next we build a scale model of the syringe with parameters Δp_m, H_m, D_m, U_m and d_m. The density ρ_m and acceleration of gravity g_m are the same as in the prototype.

5p f) Suppose the diameter of the model has been chosen to be D_m = 8 cm. Calculate H_m, U_m and d_m for the model so that the model satisfies the similarity requirements.

3p g) We use the scale model and measure Δp_m. How large is this Δp_m if the model is set up properly?

Question 3 (30p)

A Quonset hut (see left figure) is subject to a storm with horizontal speed U = 100 km/hour. The shape of the hut can be approximated by a long cylinder with a half-circular cross-section, so that a two-dimensional analysis can be used (see right figure). The diameter $2a$ of the hut is 6 m and the air has density $\rho = 1.23$ kg/m^3 and viscosity $\mu = 1.81\text{x}10^{-5}$ Pa s. Ideal flow theory will be used to analyze this problem.



3p a) Verify whether ideal flow theory can indeed be used.

4p b) What are the two main differences between ideal flows and real flows around the same object (e.g. a cylinder). Use drawings, if needed, to support your answer.

This two-dimensional problem can be analyzed using the following velocity potential $\phi\,(x,y)$:

$$\phi = Ux + \frac{Ua^2x}{x^2 + y^2}$$

2p c) This potential is a superposition of two potentials corresponding to elementary flows. Which elementary flows are these?

5p d) Rewrite $\phi\,(x,y)$ in terms of polar coordinates (r,θ) and use this to derive the corresponding stream function:

$$\psi = U\left(r - \frac{a^2}{r}\right)\sin\theta$$

$$[\, u_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta},\ u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r}].$$

2p e) Determine the location of the stagnation points (in Cartesian or polar coordinates).

2p f) The shape of the hut corresponds to a stream line that is connected to the stagnation points. What is the value of the stream function on this stagnation stream line?

7p g) Determine the pressure at points (1) and (2) on the surface of the hut. Point (1) has coordinates $(r,\theta) = (a, \pi)$ and point (2) has coordinates $(r,\theta) = (a, \pi/2)$. The velocity and pressure far upstream from the hut are U and p_{atm} (101.3 kPa), respectively. Ignore differences in height.

5p h) Sketch the pressure distribution on the surface of the hut ranging from $\theta = 0$ to $\theta = \pi$. This pressure distribution led to d'Alembert's paradox. Discuss this paradox.

N.B. For all three questions: when for some reason you are unable to answer a part of a question (a, b, etc.), make a realistic assumption and use this for the rest of the question.

GOOD LUCK!